

### Aufgabe 1

Ein Bitstring der Länge vier wird zufällig erzeugt, an jeder Position kann also mit gleicher Wahrscheinlichkeit 0 oder 1 auftreten. Es sei  $E$  das Ereignis, daß der Bitstring mit 1 beginnt. Ferner sei  $F$  das Ereignis, daß der Bitstring eine gerade Anzahl 1er enthält. Sind  $E$  und  $F$  unabhängig?

### Aufgabe 2

Was können Sie über das Ereignis  $A$  aussagen, wenn es von sich selbst unabhängig ist? Wenn die Ereignisse  $A$  und  $B$  punktfremd und unabhängig sind, was kann man dann über sie aussagen?

### Aufgabe 3

There are three prisoners in a maximum-security penitentiary for fictional villains: the Evil Wizard Voldemort, the Dark Lord Sauron, and Little Bunny Foo-Foo. The parole board has declared that it will release two of the three, chosen uniformly at random, but has not yet released their names. Naturally, Sauron figures that he will be released to his home in Mordor, where the shadows lie, with probability  $2/3$ .

A guard offers to tell Sauron the name of one of the other prisoners who will be released (either Voldemort or Foo-Foo). However, Sauron declines the offer. He reasons that if the guard says, for example, “Little Bunny Foo-Foo will be released”, then his own probability of release will drop to  $1/2$ . This is because he will then know that either he or Voldemort will also be released, and these two events are equally likely.

Using conditional probability, either prove that the Dark Lord Sauron has reasoned correctly or prove that he is wrong. Assume that if the guard has a choice of naming either Voldemort or Foo-Foo (because both are to be released), then he names one of the two uniformly at random.

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